Configuration Design Sensitivity Analysis for Dynamic Systems Using CAD-Based Velocity Field

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Nomenclature

$a_\Omega(\cdot,\cdot)$ = bilinear form due to strain energy
$a'_{V_\theta}(\cdot,\cdot)$ = variation of $a_\Omega(\cdot,\cdot)$ due to orientation change
$a'_{V_\Omega}(\cdot,\cdot)$ = variation of $a_\Omega(\cdot,\cdot)$ due to shape change
$c_\Omega(\cdot,\cdot)$ = bilinear form due to the damping
$d_\Omega(\cdot,\cdot)$ = bilinear form due to kinetic energy
$e(\cdot,\cdot)$ = bilinear mapping
h = thickness of the design component
$\ell_\Omega(\cdot)$ = load linear form
V = design velocity field
V_θ = orientation design velocity field
V_Ω = shape design velocity field
$\boldsymbol{y}$ = eigenvector of structure
Z = space of kinematically admissible displacements
z = displacement vector
$\Delta\psi$ = sensitivity coefficient from central finite difference method
ζ = eigenvalue of structure
ζ' = variation of ζ due to design change
ζ'_{V_θ} = variation of ζ due to orientation change
ζ'_{V_Ω} = variation of ζ due to shape change
$\boldsymbol{\lambda}$ = adjoint displacement vector
ρ = mass density
ς = hysteretic damping coefficient
ψ' = continuum design sensitivity coefficient
ω = natural frequency of structure
∇ = gradient operator

I. Introduction

IF the dimensions or the control points of the CAD geometry are used as the design variables in a commercial CAD tool on design, the design engineer can easily obtain the design intent. Recently, many authors have selected design variables from the parameters of CAD geometry. For example, Hardee et al.[1] developed the CAD-based shape design sensitivity analysis (DSA) of solids using Pro/ENGINEER. In this Note, CAD-based configuration DSA for dynamic systems of plates using Pro/ENGINEER is proposed. For calculation of design velocity fields, this Note uses a hybrid method that contains the CAD-based finite difference method for computation of boundary design velocity fields and boundary displacement methods for computation of domain design velocity fields.

The shape design variable is considered in a fixed coordinate system of structure; however, the configuration design variable is used to design structures with a rotational coordinate. When the continuum approach was used, Twu and Choi[2] and Wang and Choi[3] developed a configuration DSA method for static and eigenvalue problems and for transient response, respectively. Later, Park and Choi[4] extended the method to nonlinear structural systems for static problems. The orientation design velocity field is obtained using the linear velocity form. Moreover, this Note proposes CAD-based configuration DSA for dynamic problems of plates, that is, the calculation of eigenvalue and frequency-response problems for structures.

II. Parameterization and Computation of Design Velocity Field with CAD

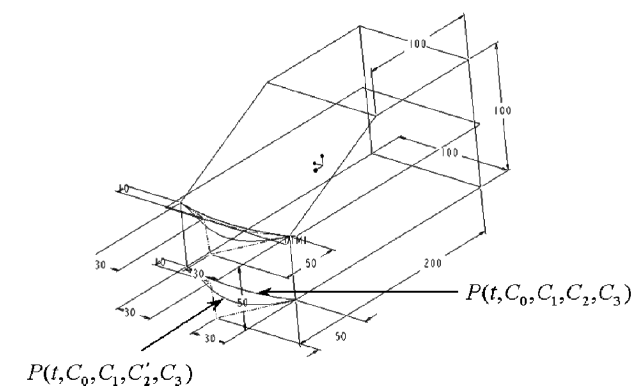
In an automated structural design optimization procedure, the designer must select design variables that specify the design of a mechanical system. The geometry of the model is parameterized by using design variables. Once a new design is obtained by the optimization algorithm, CAD geometry and the location of the nodal points must be changed based on the design parameterization. The design velocity is calculated at each surface node of the finite element mesh lying on a surface of the CAD model. For the computation of boundary design velocity, a CAD-based finite difference method is used. A CAD-based finite difference method compares the before and after of the change of CAD models. Once the boundary velocity has been calculated, an auxiliary elasticity problem is then solved, by the use of the prescribed displacements on the design boundary velocity to generate the domain design velocity field.

For example, the procedure to compute the boundary design velocity field using CAD is shown in Fig. 1. When a simplified automobile CAD model with Bezier curve and surface is considered, the boundary curve $P(t)$ of a cubic Bezier curve and the perturbed boundary curve are represented by

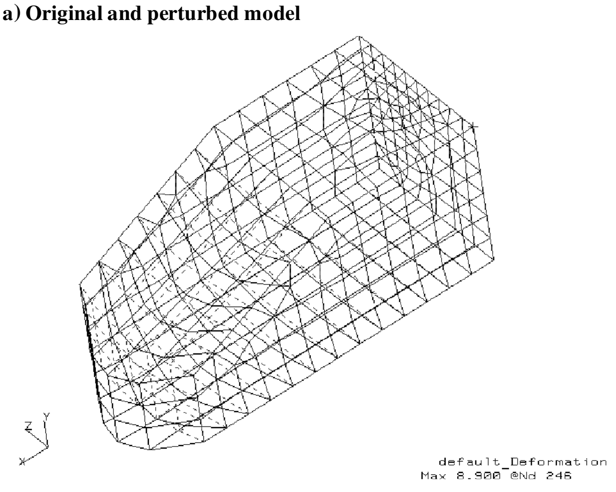
$$P(t, C_0, C_1, C_2, C_3) = C_0(1-t)^3 + 3C_1t(1-t)^2 + 3C_2t^2(1-t) + C_3t^3, \qquad 0 \le t \le 1 \tag{1}$$

$$\begin{aligned} P(t, C_0, C_1, C_2', C_3) &= P\left[t, C_0, C_1, C_2\left(x_{C_2}, y_{C_2} + \delta y_{C_2}, z_{C_2}\right), C_3\right] \\ &= C_0(1-t)^3 + 3C_1t(1-t)^2 + 3C_2\left(x_{C_2}, y_{C_2} + \delta y_{C_2}, z_{C_2}\right)t^2(1-t) + C_3t^3, \qquad 0 \le t \le 1 \end{aligned} \tag{2}$$

where C_0, C_1, C_2, and C_3 are control points of Bezier curve and $C_2' = C_2(x_{C_2}, y_{C_2} + \delta y_{C_2}, z_{C_2})$ is the perturbed point from



a) Original and perturbed model



b) Boundary and domain design velocity field

Fig. 1 Computational procedure of design velocity field (simplified automobile).

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C_2. Here δy_{C_2} is the y-directional perturbation of the control point C_2.

The design velocity at N, a node on the Bezier boundary curve, is

$$V(N) = \frac{[P(t, C_0, C_1, C_2', C_3) - P(t, C_0, C_1, C_2, C_3)]}{\delta y_{C_2}} \tag{3}$$

The design velocity field is automatically calculated in the CAD tool, Pro/ENGINEER, by using the menu bar, which is programmed using Pro/TOOLKIT in Pro/ENGINEER.

III. Configuration Design Sensitivity Analysis of Eigenvalue Problems

The variational equation of an eigenvalue problem can be written as[5]

$$a_\Omega(\boldsymbol{y}, \bar{\boldsymbol{y}}) = \zeta d_\Omega(\boldsymbol{y}, \bar{\boldsymbol{y}}) \tag{4}$$

for all $\bar{\boldsymbol{y}} \in Z$, where the eigenvalue $\zeta = \omega^2$. The energy bilinear form $a_\Omega(\boldsymbol{y}, \bar{\boldsymbol{y}})$ and

$$d_\Omega(\boldsymbol{y}, \bar{\boldsymbol{y}}) = \iint_\Omega e(\boldsymbol{y}, \bar{\boldsymbol{y}})\, \mathrm{d}\Omega$$

are strain and kinetic energy bilinear forms, respectively, and $e(\cdot, \cdot)$ is a bilinear mapping. Because the eigenvector $\boldsymbol{y}$ is orthonormal relative to the mass matrix, a normalizing condition must be used to define the eigenfunction uniquely. The normalizing condition is

$$d_\Omega(\boldsymbol{y}, \boldsymbol{y}) = 1 \tag{5}$$

The first variation of Eq. (4), for all $\bar{\boldsymbol{y}} \in Z$, is

$$\begin{aligned} [a_\Omega(\boldsymbol{y}, \bar{\boldsymbol{y}})]' &\equiv a_\Omega(\dot{\boldsymbol{y}}, \bar{\boldsymbol{y}}) + a_\Omega(\boldsymbol{y}, \dot{\bar{\boldsymbol{y}}}) + a'_{V_\Omega}(\boldsymbol{y}, \bar{\boldsymbol{y}}) + a'_{V_\theta}(\boldsymbol{y}, \bar{\boldsymbol{y}}) \\ &= \left(\zeta'_{V_\Omega} + \zeta'_{V_\theta}\right) d_\Omega(\boldsymbol{y}, \bar{\boldsymbol{y}}) + \zeta [d_\Omega(\dot{\boldsymbol{y}}, \bar{\boldsymbol{y}}) + d_\Omega(\boldsymbol{y}, \dot{\bar{\boldsymbol{y}}}) \\ &\quad + d'_{V_\Omega}(\boldsymbol{y}, \bar{\boldsymbol{y}}) + d'_{V_\theta}(\boldsymbol{y}, \bar{\boldsymbol{y}})] \equiv \zeta' d_\Omega(\boldsymbol{y}, \bar{\boldsymbol{y}}) + \zeta [d_\Omega(\boldsymbol{y}, \bar{\boldsymbol{y}})]' \end{aligned} \tag{6}$$

where $\zeta' = \zeta'_{V_\Omega} + \zeta'_{V_\theta}$, $a(\dot{\boldsymbol{y}}, \bar{\boldsymbol{y}}) = \zeta d(\dot{\boldsymbol{y}}, \bar{\boldsymbol{y}})$, and $a(\boldsymbol{y}, \dot{\bar{\boldsymbol{y}}}) = \zeta d(\boldsymbol{y}, \dot{\bar{\boldsymbol{y}}})$ for all $\dot{\bar{\boldsymbol{y}}} \in Z$. Because Eq. (6) holds for all $\bar{\boldsymbol{y}} \in Z$, this equation may be evaluated with $\bar{\boldsymbol{y}} = \boldsymbol{y}$. When the normalizing condition is used, Eq. (6) is rewritten as

$$\begin{aligned} \zeta' &= [a_\Omega(\boldsymbol{y}, \boldsymbol{y})]' - \zeta [d_\Omega(\boldsymbol{y}, \boldsymbol{y})]' \\ &\equiv a'_{V_\Omega}(\boldsymbol{y}, \boldsymbol{y}) + a'_{V_\theta}(\boldsymbol{y}, \boldsymbol{y}) - \zeta \left[d'_{V_\Omega}(\boldsymbol{y}, \boldsymbol{y}) + d'_{V_\theta}(\boldsymbol{y}, \boldsymbol{y})\right] \end{aligned} \tag{7}$$

The first variation of the bilinear form due to mass effect is

$$\begin{aligned} [d_\Omega(z, \bar{z})]' &= d_\Omega(\dot{z}, \bar{z}) - d_\Omega\left(\nabla z^T V_\Omega, \bar{z}\right) - d_\Omega(\tilde{V}_\theta z, \bar{z}) \\ &\quad + d_\Omega(z, \dot{\bar{z}}) - d_\Omega\left(z, \nabla \bar{z}^T V_\Omega\right) - d_\Omega(z, \tilde{V}_\theta \bar{z}) + d^0_\Omega(z, \bar{z}) \\ &= d_\Omega(\dot{z}, \bar{z}) + d_\Omega(z, \dot{\bar{z}}) + d'_{V_\Omega}(z, \bar{z}) + d'_{V_\theta}(z, \bar{z}) \end{aligned} \tag{8}$$

where

$$d^0_\Omega(z, \bar{z}) = \iint_\Omega \mathrm{div}[e(z, \bar{z}) V_\Omega]\, \mathrm{d}\Omega$$

$$\dot{z}_{V_\Omega} = z'_{V_\Omega} + \nabla z^T V_\Omega, \qquad \dot{z}_{V_\theta} = z'_{V_\theta} + \tilde{V}_\theta z \tag{9}$$

When the linear velocity form is used, the orientation design velocity $\tilde{V}_\theta$ does not contain any coupled terms.[4] The differentials $d'_{V_\Omega}(z, \bar{z})$ and $d'_{V_\theta}(z, \bar{z})$ in Eq. (8) denote the explicit dependence of kinetic energy form due to the shape and orientation changes shown as

$$\begin{aligned} d'_{V_\Omega}(z, \bar{z}) &= -d_\Omega\left(\nabla z^T V_\Omega, \bar{z}\right) - d_\Omega\left(z, \nabla \bar{z}^T V_\Omega\right) + d^0_\Omega(z, \bar{z}) \\ d'_{V_\theta}(z, \bar{z}) &= -d_\Omega(\tilde{V}_\theta z, \bar{z}) - d_\Omega(z, \tilde{V}_\theta \bar{z}) \end{aligned} \tag{10}$$

Consider the plane elastic plate design component. The bilinear form due to mass effects is

$$d_\Omega(z, \bar{z}) = \iint_\Omega \rho h z^T \bar{z}\, \mathrm{d}\Omega = \iint_\Omega \rho h (z_1 \bar{z}_1 + z_2 \bar{z}_2 + z_3 \bar{z}_3)\, \mathrm{d}\Omega \tag{11}$$

When Eq. (11) is used, the first variations of bilinear form $d_\Omega(z, \bar{z})$ in Eq. (10) become

$$d'_{V_\Omega}(z, \bar{z}) = \iint_\Omega [\rho h (z_1 \bar{z}_1 + z_2 \bar{z}_2 + z_3 \bar{z}_3)]\, \mathrm{div} V_\Omega\, \mathrm{d}\Omega, \qquad d'_{V_\theta}(z, \bar{z}) = 0 \tag{12}$$

From Eq. (7), the eigenvalue design sensitivity expression of the plane elastic plate is given as

$$\begin{aligned} \zeta' = [a_\Omega(\boldsymbol{y}, \boldsymbol{y})]' - \zeta [d_\Omega(\boldsymbol{y}, \boldsymbol{y})]' &\equiv a'_{V_\Omega}(\boldsymbol{y}, \boldsymbol{y}) \\ &\quad + a'_{V_\theta}(\boldsymbol{y}, \boldsymbol{y}) - \zeta \left[d'_{V_\Omega}(\boldsymbol{y}, \boldsymbol{y})\right] \end{aligned} \tag{13}$$

where the computation of the first variation of strain energy bilinear form $[a_\Omega(\boldsymbol{y}, \boldsymbol{y})]'$ is similar to static problem[2,4] and all terms on the right-hand side are explicit.

IV. Configuration DSA of Dynamic Frequency-Response Problem

The variational equation of a dynamic frequency-response problem can be written as

$$b_\Omega(z, \bar{z}) \equiv -\omega^2 \rho d_\Omega(z, \bar{z}) + i\omega c_\Omega(z, \bar{z}) + a_\Omega(z, \bar{z}) = \ell_\Omega(\bar{z}) \tag{14}$$

for all $\bar{z} \in Z$, where $c_\Omega(z, \bar{z})$ is the bilinear form due to the damping and $\ell_\Omega(\bar{z})$ is the load linear form.

Similarly, in the eigenvalue case, the first variation of Eq. (14) is

$$\begin{aligned} [b_\Omega(z, \bar{z})]' &= b_\Omega\left(\dot{z}_{V_\Omega}, \bar{z}\right) + b_\Omega\left(\dot{z}_{V_\theta}, \bar{z}\right) + b_\Omega\left(z, \dot{\bar{z}}_{V_\Omega}\right) + b_\Omega(z, \dot{\bar{z}}_{V_\theta}) \\ &\quad - b_\Omega\left(\nabla z^T V_\Omega, \bar{z}\right) - b_\Omega(\tilde{V}_\theta z, \bar{z}) - b_\Omega\left(z, \nabla \bar{z}^T V_\Omega\right) \\ &\quad - b_\Omega(z, \tilde{V}_\theta \bar{z}) + b^0_\Omega(z, \bar{z}) = \ell'_{V_\Omega}(\bar{z}) + \ell'_{V_\theta}(\bar{z}) = [\ell_\Omega(\bar{z})]' \end{aligned} \tag{15}$$

Next, consider a general functional that may be written in integral form as

$$\psi = \iint_\Omega g(z, \nabla z)\, \mathrm{d}\Omega \tag{16}$$

where $\nabla z = [\nabla z_1 \nabla z_2 \nabla z_3]^T$ and the function g is continuously differentiable with respect to its arguments. The variation of the functional of Eq. (16) is

$$\begin{aligned} \psi' = \iint_\Omega &\left[g_z \dot{z} + g_{\nabla z} \nabla \dot{z} - g_z\left(\nabla z^T V_\Omega + \tilde{V}_\theta z\right)\right. \\ &\left. - g_{\nabla z} \nabla\left(\nabla z^T V_\Omega + \tilde{V}_\theta z\right) + \nabla g^T V_\Omega + g\, \mathrm{div} V_\Omega\right] \mathrm{d}\Omega \end{aligned} \tag{17}$$

where $g_{\nabla z} = [\partial g/\partial z_1\ \partial g/\partial z_2\ \partial g/\partial z_3]$ and $\dot{z}$ and $\nabla \dot{z}$ depend on the velocity field. The objective here is to obtain an explicit expression for ψ' in terms of the velocity field, which requires rewriting the first two terms of the last integral on the right-hand side of Eq. (17) explicitly in terms of velocity, that is, eliminating $\dot{z}$. To eliminate $\dot{z}$, an adjoint equation is introduced by replacing $\dot{z} \in Z$ by a virtual displacement $\bar{\boldsymbol{\lambda}} \in Z$ and equating the sum of terms involving $\bar{\boldsymbol{\lambda}}$ to the bilinear form,[5]

$$b_\Omega(\boldsymbol{\lambda}, \bar{\boldsymbol{\lambda}}) = \iint_\Omega [g_z \bar{\boldsymbol{\lambda}} + g_{\nabla z} \nabla \bar{\boldsymbol{\lambda}}]\, \mathrm{d}\Omega \tag{18}$$

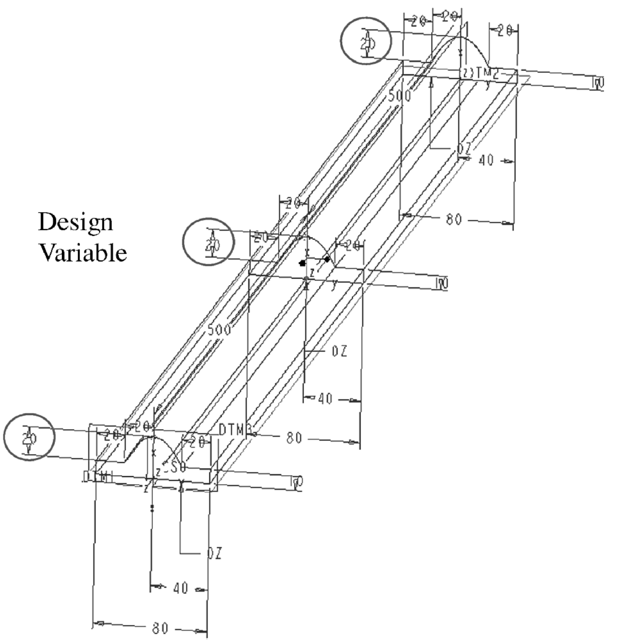
for all $\bar{\boldsymbol{\lambda}} \in Z$. When the adjoint equation (18) is used, Eq. (17) becomes

$$\begin{aligned} \psi' &= \ell'_{V_\Omega}(\boldsymbol{\lambda}) + \ell'_{V_\theta}(\boldsymbol{\lambda}) - b'_{V_\Omega}(z, \boldsymbol{\lambda}) - b'_{V_\theta}(z, \boldsymbol{\lambda}) \\ &\quad - \iint_\Omega \left[g_z\left(\nabla z^T V_\Omega\right) + g_z(\tilde{V}_\theta z) + g_{\nabla z} \nabla\left(\nabla z^T V_\Omega\right)\right. \\ &\quad \left. + g_{\nabla z} \nabla(\tilde{V}_\theta z)\right] \mathrm{d}\Omega + \iint_\Omega \left[\nabla g^T V_\Omega + g\, \mathrm{div} V_\Omega\right] \mathrm{d}\Omega \end{aligned} \tag{19}$$

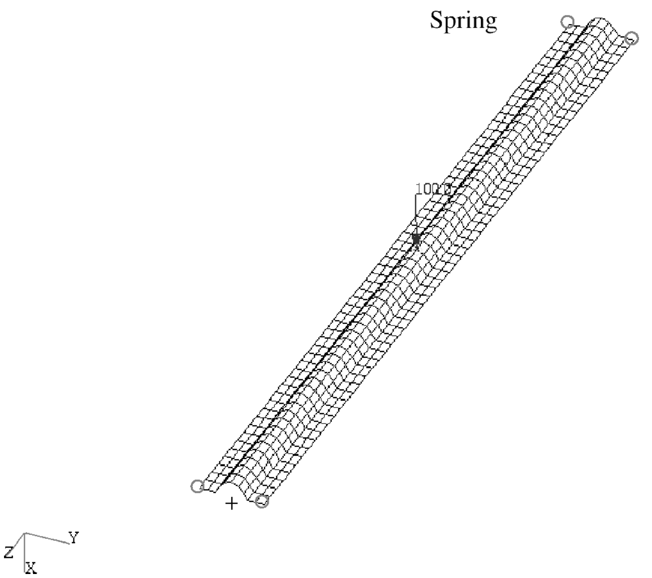
where $\boldsymbol{\lambda}$ are determined as solutions of the adjoint equation.

Table 1 Sensitivity verification for eigenvalue and vibration (displacement)

Frequency response, Hz	ψ	ψ'	$\Delta\psi$ (0.01)	Error $(\Delta\psi - \psi')/\Delta\psi \times 100\%$, %
		Eigenvalue		
First bending mode	$6.660069E{+}5$	$4.587973E{+}4$	$4.639000E{+}4$	1.10
		Vibration (x-direction displacement at node 552)		
126	$4.237094E{-}3$	$-5.243227E{-}3$	$-5.263800E{-}3$	0.39
127	$5.693647E{-}3$	$-9.040565E{-}3$	$-9.069300E{-}3$	0.32
128	**$8.491452E{-}3$**	**$-1.885771E{-}2$**	**$-1.885400E{-}2$**	**−0.02**
129	$1.530894E{-}2$	$-5.100493E{-}2$	$-5.009250E{-}2$	−1.82
130	$2.486804E{-}2$	$2.475327E{-}2$	$2.308200E{-}2$	−7.24
131	$1.341361E{-}2$	$3.833328E{-}2$	$3.808900E{-}2$	−0.64
132	**$7.960017E{-}3$**	**$1.449424E{-}2$**	**$1.461290E{-}2$**	**0.81**
133	$5.587468E{-}3$	$7.092731E{-}3$	$7.185450E{-}3$	1.29
134	$4.299933E{-}3$	$4.110735E{-}3$	$4.177000E{-}3$	1.59



a) CAD model and design variable



b) Finite element model of vibration analysis

Fig. 2 Reinforcement model.

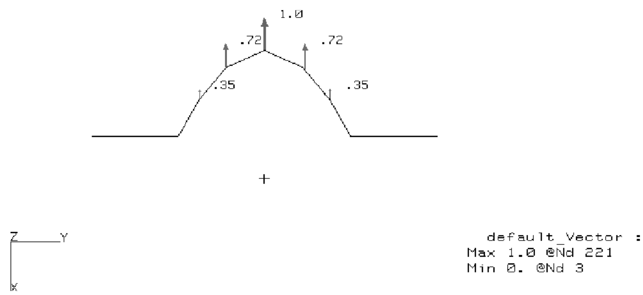


Fig. 3 Design velocity field of reinforcement model.

For the hysteretic damping case, Eq. (14) can be written as

$$b_\Omega(z,\bar{z}) = -\omega^2 \rho d_\Omega(z,\bar{z}) + (1+i\varsigma)a_\Omega(z,\bar{z}) = \ell_\Omega(\bar{z}) \tag{20}$$

for all $\bar{z} \in Z$. Thus, Eq. (15) is rewritten as

$$[b_\Omega(z,\bar{z})]' = -\omega^2 \rho [d_\Omega(z,\bar{z})]' + (1+i\varsigma)[a_\Omega(z,\bar{z})]' = [\ell_\Omega(\bar{z})]' \tag{21}$$

for all $\bar{z} \in Z$.

V. Numerical Examples

The geometry with conic surface and finite element model of a reinforcement shown in Fig. 2 are created using Pro/ENGINEER. The finite element model has 580 shell elements, 12 spring elements, and 649 nodes. The height of the reinforcement model is selected as the configuration design variable. The boundary velocity field is obtained by using Pro/ENGINEER, and the domain velocity field is calculated using the boundary displacement method in MSC/NASTRAN as shown in Fig. 3.

For eigenvalue problem, the first bending mode of this model occurs at 129.9 Hz. The design sensitivity coefficient of the first bending mode with respect to the height as design variable is computed using a continuum DSA program (SENS developed by the authors).

The boundary conditions and a load for vibration analysis are shown in Fig. 2. The model is supported by three springs for each corner node. A force of 100 magnitudes is applied at a center node. The frequency response at the center of the reinforcement is evaluated between 0 and 200 Hz and has two peaks at 128 and 132 Hz, indicated by the boldfaced type in Table 1. DSA of frequency responses from 126 to 134 Hz are carried out with respect to configuration design variables based on the height of the reinforcement model. The accuracy of computed sensitivity coefficients are verified using the central finite difference method (CFDM). In Table 1, ψ' is the continuum design sensitivity coefficient and $\Delta\psi$ is the sensitivity coefficient from CFDM. Table 1 shows that the sensitivity coefficients from SENS are accurate.

VI. Conclusions

A configuration DSA for dynamic systems with a plate is developed by using CAD parameters selected from CAD geometry in a commercial CAD tool. With this system, the designer can easily obtain configuration design velocity fields using Pro/ENGINEER and Pro/TOOLKIT. The numerical example of a reinforcement model shows that the proposed configuration DSA of eigenvalue and frequency-response results of plate are accurate.

Acknowledgment

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Hybrid Reanalysis Method for Eigenproblems of Topological Modifications

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Introduction

STRUCTURAL optimization has been the subject of numerous studies in recent years.[1–6] Topological modifications can greatly improve a typical design; however, the solutions of topological optimization problems are difficult because of changes in the structural model. In particular, changes in the number of variables and degrees of freedom result in corresponding changes in the form of the analysis equation.

One of the main obstacles in topological modification analysis is the high computational effort involved in repeated analysis. As structural systems to be solved for static and dynamic characteristics become larger, the computing time and the corresponding cost increase drastically. Hence, various techniques have been used to reduce the size of the system or the dimensions of the structural matrices involved in the formulation.[7–12] The reduction schemes increase the calculation efficiency at the expense of solution accuracy.

In previous studies, two sets of degrees of freedom (DOF), called secondary and primary, are introduced in repeated analysis. During the solution, the secondary set is condensed out, whereas the primary one is retained. When the transformation matrix derived from the stiffness and mass matrix is used, the system to be solved is transformed into a reduced subspace represented by the primary degrees of freedom. An important problem concerns which DOF should go into the primary set. Improper selection may not only result in missing some of the lowest modes but also cause difficulties in programming because one must redecompose stiffness and mass matrices according to the selected primary and secondary sets.

Considering that the secondary and primary DOF method is used commonly in substructures, we intend to introduce this method into dynamic reanalysis of topological modification. In the present study, the DOF in the initial system are selected as the primary set whereas the ones added in the modified system are selected as the secondary set. When static condensation and Rayleigh quotient are used and the effects of the mass added in the modified system are considered, several eigenpairs are obtained simultaneously. The results show that the proposed method can give high accuracy.

Problem Formulation

We consider only the case where both the design variables and the number of DOF are added in the modified system. In this case, the generalized eigenproblem is as follows:

$$\boldsymbol{K}\boldsymbol{V} = \lambda \boldsymbol{M}\boldsymbol{V} \tag{1}$$

where

$$\boldsymbol{K} = \boldsymbol{K}_0' + \Delta\boldsymbol{K}' \tag{2}$$

$$\boldsymbol{M} = \boldsymbol{M}_0' + \Delta\boldsymbol{M}' \tag{3}$$

$$\boldsymbol{K}_0' = \begin{bmatrix} \boldsymbol{K}_0 & 0 \\ 0 & 0 \end{bmatrix} \tag{4}$$

$$\boldsymbol{M}_0' = \begin{bmatrix} \boldsymbol{M}_0 & 0 \\ 0 & 0 \end{bmatrix} \tag{5}$$

$$\Delta\boldsymbol{K}' = \begin{bmatrix} \Delta\boldsymbol{K}_{nn} & \Delta\boldsymbol{K}_{nm} \\ \Delta\boldsymbol{K}_{mn} & \Delta\boldsymbol{K}_{mm} \end{bmatrix} \tag{6}$$

$$\Delta\boldsymbol{M}' = \begin{bmatrix} \Delta\boldsymbol{M}_{nn} & \Delta\boldsymbol{M}_{nm} \\ \Delta\boldsymbol{M}_{mn} & \Delta\boldsymbol{M}_{mm} \end{bmatrix} \tag{7}$$

$\boldsymbol{K}_0$ and $\boldsymbol{M}_0$ are the stiffness and mass matrices of the initial structure, respectively. Subscript n denotes the number of DOF of the initial structure and m the augmentation of the DOF of the modified structure. If the DOF in the initial structure are selected as the primary set and the ones added in the modified structure are selected as the secondary one, having assembled the change of stiffness and mass matrices for the added new nodes and members, from Eqs. (1–7), it can be seen that the stiffness and mass matrices do not have to be redecomposed.

Proposed Method

Substituting Eqs. (2–7) into Eq. (1) yields

$$\begin{bmatrix} \boldsymbol{K}_0 + \Delta\boldsymbol{K}_{nn} & \Delta\boldsymbol{K}_{nm} \\ \Delta\boldsymbol{K}_{mn} & \Delta\boldsymbol{K}_{mm} \end{bmatrix} \begin{pmatrix} \boldsymbol{V}_n \\ \boldsymbol{V}_m \end{pmatrix} = \lambda \begin{bmatrix} \boldsymbol{M}_0 + \Delta\boldsymbol{M}_{nn} & \Delta\boldsymbol{M}_{nm} \\ \Delta\boldsymbol{M}_{mn} & \Delta\boldsymbol{M}_{mm} \end{bmatrix} \begin{pmatrix} \boldsymbol{V}_n \\ \boldsymbol{V}_m \end{pmatrix} \tag{8}$$

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